

# Structural vertices of extended $SU(3)$ -chiral lagrangians in the large- $N_c$ approach<sup>1</sup>

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We establish the connection between the structural coupling constants of the phenomenological chiral lagrangian and the coefficients of effective lagrangians obtained in the QCD-bosonization models by means of the derivative expansion. The extension of the chiral lagrangian describing matrix elements of the pseudoscalar gluon density is examined. The large- $N_c$  relations for corresponding structural constants are elaborated.

## 1. Introduction

The chiral lagrangians for light  $SU(3)_f$ -pseudoscalar mesons [1, 2] describe their strong and electroweak interactions at low energies with reasonable precision when the Chiral Perturbation Theory (CPT) is applied (see[2, 3, 4] and references therein). The modern chiral lagrangians reflect the exact, softly broken and spontaneously broken symmetries of the Quantum Chromodynamics (QCD) and include the interaction vertices compatible with these symmetries. The corresponding coupling constants contain the detailed information about the QCD dynamics and have not been yet calculated from the QCD in a reliable way. Practically they are established from the phenomenology [2, 3, 4].

On the other hand the QCD-bosonization approaches [5, 6] and the quark models [7, 8, 9] have been developed to calculate the interaction vertices of the  $SU(3)$ -chiral lagrangian. When following the large- $N_c$  counting in the QCD one can derive the general form of the chiral lagrangian which is characteristic for any bosonization model. This "model-framework" lagrangian is obtained from the bosonization of one-quark loop by application of the derivative expansion and does not coincide with the phenomenological one in some vertices. Therefore it is of real interest to establish the relations between them in order to reveal new constraints on the phenomenological constants which can be checked in experimental data.

The main goal of this paper is to elaborate the model-framework parametrization of effective coupling constants of the extended chiral lagrangian which is suitable for the description of the low-energy matrix elements of vector, axial-vector, scalar and pseudoscalar currents[2] as well as of the matrix elements of the pseudoscalar gluon density[10]. On this way we find the new set of OZI rules. In particular, one of them predicts the branching ratio of the decays  $\psi' \rightarrow J/\psi + \pi$  or  $\eta$  [10].

## 2. $SU(3)$ -chiral lagrangian to the $p^4$ -order

The phenomenological chiral lagrangian for  $SU(3)$  octet of pseudoscalar meson fields was elaborated in the  $p^2 + p^4$ -order [1, 2] of CPT with taking into account of basic symmetries

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of the QCD. This lagrangian has the following structure,

$$S_{eff} = \int d^4x (\mathcal{L}_2 + \mathcal{L}_4) + S_{WZW}, \quad (1)$$

The Weinberg lagrangian is given by

$$\mathcal{L}_2 = \frac{F_0^2}{4} [ < (D_\mu U)^\dagger D^\mu U > + < \chi^\dagger U + U^\dagger \chi > ]. \quad (2)$$

where  $F_0 \simeq 88\text{MeV}$  is a bare pion-decay constant,  $U$  is a  $SU(3)$ -chiral field describing pseudoscalar mesons,  $UU^\dagger = 1$ ,  $\det U = 1$ . The external sources are accumulated in the covariant derivative,  $D_\mu = \partial_\mu + [V_\mu, *] + \{A_\mu, *\}$  with vector,  $V_\mu$  and axial-vector,  $A_\mu$  fields and in the complex density,  $\chi = 2B_0(s+ip)$  with scalar and pseudoscalar fields. The constant  $B_0$  is related to the scalar quark condensate, the order parameter of the spontaneous chiral symmetry breaking in the QCD.

The Gasser-Leutwyler lagrangian contains ten basic low-energy constants  $L_i$ ,

$$\begin{aligned} \mathcal{L}_4^{GL} = & L_1 < (D_\mu U)^\dagger D^\mu U >^2 + L_2 < (D_\mu U)^\dagger D_\nu U \times (D^\mu U)^\dagger D^\nu U > \\ & + L_3 < (D^\mu U)^\dagger D_\mu U (D^\nu U)^\dagger D_\nu U > + L_4 < (D^\mu U)^\dagger D_\mu U > < \chi^\dagger U + U^\dagger \chi > \\ & + L_5 < (D^\mu U)^\dagger D_\mu U (\chi^\dagger U + U^\dagger \chi) > + L_6 < \chi^\dagger U + U^\dagger \chi >^2 \\ & + L_7 < \chi^\dagger U - U^\dagger \chi >^2 + L_8 < \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi > \\ & + L_9 < F_{\mu\nu}^R D^\mu U (D^\nu U)^\dagger + F_{\mu\nu}^L (D^\mu U)^\dagger D^\nu U > + L_{10} < U^\dagger F_{\mu\nu}^R U F^{L\mu\nu} > \end{aligned} \quad (3)$$

where  $F_{\mu\nu}^L = \partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu]$  and  $L_\mu = V_\mu + A_\mu$ ;  $R_\mu = V_\mu - A_\mu$ . The Wess-Zumino-Witten action  $S_{WZW}$  assembles so-called anomalous vertices [2, 3] and it is not displayed here (see the next Sec.).

The derivation of the chiral lagrangian from the QCD by direct bosonization[5] or by bosonization of a quark model [7, 8, 9] can be described schematically as follows,

$$\int \mathcal{D}G \mathcal{D}q \mathcal{D}\bar{q} \exp(-S(\bar{q}, q, G; V, A, s, p)) = \int \mathcal{D}U \exp(-S_{eff}(U; V, A, s, p) \cdot \mathcal{R}). \quad (4)$$

where the averaging over gluons,  $G_\mu$ , and/or quarks,  $q, \bar{q}$  is approximately replaced by the averaging over chiral fields  $U$  and then the effective action is expanded in derivatives of chiral and external fields. The model design consists in the choice of model parameters (the type and size of the momentum cutoff, the spectral asymmetry or the dynamical mass etc.) and in the assumption that the remainder  $\mathcal{R} \simeq 1$  at low energies.

In the large- $N_c$  approach the main contribution into the effective lagrangian  $\mathcal{L}_4$  is parametrized by nine structural constants,  $I_k$

$$\begin{aligned} \mathcal{L}_4^{eff} = & I_1 < D_\mu U (D_\nu U)^\dagger D^\mu U (D^\nu U)^\dagger > + I_2 < D_\mu U (D_\mu U)^\dagger D^\nu U (D^\nu U)^\dagger > \\ & + I_3 < (D_\mu^2 U)^\dagger D_\nu^2 U > + I_4 < (D_\mu \chi)^\dagger D^\mu U + D_\mu \chi (D^\mu U)^\dagger > \\ & + I_5 < D_\mu U (D^\mu U)^\dagger (\chi U^\dagger + U \chi^\dagger) > + I_6 < U \chi^\dagger U \chi^\dagger + \chi U^\dagger \chi U^\dagger > \\ & + I_7 < \chi^\dagger U - U^\dagger \chi >^2 \\ & + I_8 < F_{\mu\nu}^R D^\mu U (D^\nu U)^\dagger + F_{\mu\nu}^L (D^\mu U)^\dagger D^\nu U > + I_9 < U^\dagger F_{\mu\nu}^R U F^{L\mu\nu} > \end{aligned} \quad (5)$$

The constants  $I_1, \dots, I_6, I_8, I_9$  arise from one-loop quark diagrams in the soft-momentum expansion (or, equivalently, from the large-mass expansion of heavy-meson lagrangians [11]) where all pole contributions of  $SU(3)$ -pseudoscalar mesons are amputated. Thereby  $I_k = O(N_c)$ ;  $k \neq 7$ . The coefficient  $I_7$  is essentially saturated by the vacuum pseudoscalar configurations [12] and it is estimated in the next Sec. The vertices  $I_3, I_4$  induce the off-shell momentum dependence of the decay constant of pseudoscalar mesons that finds a firm explanation due to existence of highly excited pseudoscalar mesons [14] ("radial excitations").

The relations between coefficients  $L_i$  and  $I_j$  are derived when one imposes the equations of motion from the Weinberg lagrangian  $\mathcal{L}_2$  following the scheme of CPT<sub>h</sub>,

$$(D_\mu^2 U)^\dagger U - U^\dagger D_\mu^2 U - \chi^\dagger U + U^\dagger \chi = \frac{1}{3} \langle U^\dagger \chi - \chi^\dagger U \rangle, \quad (6)$$

and the identities for  $SU(3)$ -chiral currents,  $A_\mu = U^\dagger \partial_\mu U$  are applied [2],

$$\langle A_\mu A_\nu A_\mu A_\nu \rangle = -2 \langle A_\mu^2 A_\nu^2 \rangle + \frac{1}{2} \langle A_\mu^2 \rangle^2 + \langle A_\mu A_\nu \rangle^2 \quad (7)$$

They read,

$$\begin{aligned} 2L_1 &= L_2 = I_1, & L_3 &= I_2 + I_3 - 2I_1, & L_4 &= L_6 = 0, & L_5 &= I_4 + I_5, \\ L_7 &= I_7 - \frac{1}{6}I_4 + \frac{1}{12}I_3, & L_8 &= -\frac{1}{4}I_3 + \frac{1}{2}I_4 + I_6, & L_9 &= I_8, & L_{10} &= I_9. \end{aligned} \quad (8)$$

From these relations one can see that seven phenomenological constants  $L_i$ ,  $i \neq 2, 4, 6$  are independent in the large- $N_c$  approximation and they are parametrized by nine model parameters  $I_j$ . Therefore there exists the two-parameter family of models which yield the same effective lagrangian for  $SU(3)$ -pseudoscalar mesons in external  $V, A, s, p$  fields. In particular one could select out  $I_3 = I_4 = 0$  as it is conventionally accepted. On the other hand the latter ones are determined by the mixing of light pseudoscalar mesons with their excitations [14].

We remark that the Kaplan-Manohar freedom under the quark mass reparametrization [10] is fixed here by the choice of  $L_6 = 0$  compatible with the large- $N_c$  estimations.

### 3. $U_A(1)$ -extension of the chiral lagrangian

Let us perform the extension of the  $SU(3)$ -chiral lagrangian which describes the bosonization of flavor-singlet currents and of the pseudoscalar gluon density. As it is well known [3, 4] the straightforward  $U(1)$ -extension of the  $SU(3)$ -lagrangian describes correctly the properties of pseudoscalar meson  $\eta'$  if one takes into account the strong vacuum effects due to nontrivial topological susceptibility [12].

In order to describe the meson matrix elements of pseudoscalar gluon density we follow [10] and supplement the QCD lagrangian with the singlet pseudoscalar source of this density,

$$\mathcal{L}_\theta = -\theta(x)G\tilde{G}; \quad \text{where} \quad G\tilde{G} \equiv \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}. \quad (9)$$

After the  $SU(3)$ -bosonization in accordance to (4) one arrives [10] to the lagrangian (1), (2), (3) with modified  $\tilde{\chi} = \chi \exp(i\theta(x)/3)$  and to the additional structural vertices  $L_i$ ,  $i = 14, \dots, 19$ ,

$$\begin{aligned}\mathcal{L}_4^{(D\theta)} &= iL_{14}D_\mu D^\mu \theta < \tilde{\chi}^\dagger U - U^\dagger \tilde{\chi} > + iL_{15}D_\mu \theta < (D^\mu \tilde{\chi})^\dagger U - U^\dagger (D^\mu \tilde{\chi}) > \\ &+ L_{16}D_\mu \theta D^\mu \theta < D_\nu U (D^\nu U)^\dagger > + L_{17}D_\mu \theta D_\nu \theta < D^\mu U (D^\nu U)^\dagger > \\ &+ L_{18}D_\mu \theta D^\mu \theta < \tilde{\chi} U^\dagger + U \tilde{\chi}^\dagger > + iL_{19}D_\mu \theta < U (D^\mu U)^\dagger D_\nu U (D^\nu U)^\dagger > .\end{aligned}\quad (10)$$

Herein the equations of motion (6) have been imposed. In comparison to [10] the vertex with  $L_{19}$  is added. It survives in the chiral limit  $m_q \rightarrow 0$  and may be important in the Skyrmin physics.

Let us consider now the  $SU(3)$ -bosonization in the model framework which brings the chiral lagrangian (10). It consists typically of the following stages. At first, one extends the  $SU(3)$ -bosonization to the  $U(3)$  one and introduce the collective pseudoscalar variables,  $U(x) \rightarrow \tilde{U} = U \exp(i\eta_0/3)$ . Then after the bosonization of one quark-loop action one reveals in general two relevant vertices describing the dynamics of singlet pseudoscalar bound state at leading order of the derivative expansion,

$$\mathcal{L}_-(\eta_0) = \left( \eta_0 + i\xi < \chi^\dagger \tilde{U} - \tilde{U}^\dagger \chi > \right) G\tilde{G}. \quad (11)$$

Evidently  $\xi = O(1)$  at large  $N_c$ . When combining this functional with the vertex (9) we see that the external source  $\theta(x)$  can be reabsorbed into the field  $\eta_0$  by shifting  $\eta_0 \rightarrow \eta_0 - \theta$ . As a result the essential part of the lagrangian (10) (except for the vertex  $L_{14}$ ) is created directly from the model framework one(5),

$$\begin{aligned}L_{15} &= -6L_{18} = -\frac{2}{3}(I_4 + I_5) = -\frac{2}{3}L_5 = (-0.9 \pm 0.3) \cdot 10^{-3}; \\ L_{16} &= \frac{1}{2}L_{17} = -\frac{1}{6}L_{19} = \frac{2}{9}(I_1 + I_2 + I_3) = \frac{2}{9}(3L_2 + L_3) = (0 \pm 1.5) \cdot 10^{-3},\end{aligned}\quad (12)$$

after the appropriate rotation  $\tilde{U} = U \exp[i(\eta_0 - \theta)/3]$ , the redefinition  $\tilde{\chi} = \chi \exp(i\theta(x)/3)$  and the elimination of heavy singlet field  $\eta_0$  in the soft-momentum limit. The numerical estimations for the coefficients  $L_2$ ,  $L_3$ ,  $L_5$  in (12) are taken from [3].

The next step is to obtain the effective vertices for  $\eta_0$  which are substantially induced by the QCD vacuum effects leading to the nontrivial correlator at zero momenta,

$$M_0^4 = - \int d^4x < 0 | T(G\tilde{G}(x)G\tilde{G}(0)) | 0 >_0, \quad (13)$$

in accordance to the common solution of  $U(1)$ -problem [12].

After averaging over gluons we derive the additional contribution into the  $U(3)$  meson lagrangian,

$$\tilde{\mathcal{L}}(\eta_0) = -\frac{M_0^4}{2} \left( \eta_0 + i\xi < \chi^\dagger \tilde{U} - \tilde{U}^\dagger \chi > \right)^2 \quad (14)$$

Since  $M_0$  is not a soft chiral parameter one can apply the large-mass reduction for  $\eta_0$  field and expand in powers of  $1/M_0$ . On this way the  $SU(3)$ -chiral lagrangian is saturated in the

vertices  $L_7, L_{14}$ ,

$$I_7 = -\frac{1}{2M_0^4} \left( \frac{F_0^2}{12} - \xi M_0^4 \right)^2; \quad L_7 = -\frac{F_0^4}{288M_0^4} - \frac{1}{6}I_4 + \frac{1}{12}I_3 + \frac{\xi F_0^2}{12} - \frac{\xi^2 M_0^4}{2}. \quad (15)$$

where the first term in  $L_7$  is of order  $O(N_c^2)$ ,  $N_c = 3$  and the next three ones are  $O(N_c)$ . Respectively, the coefficient

$$L_{14} = \frac{F_0^4}{72M_0^4} - \frac{1}{3}I_4 - \frac{2}{3}I_5 - \frac{\xi F_0^2}{6} = -4L_7 + O(N_c), \quad (16)$$

is remarkably related to the structural constant  $L_7$  in the main large- $N_c$  order. The latter one is well estimated from the  $\eta, \eta'$ -meson mass spectrum[3],  $L_7 = (-0.4 \pm 0.2) \cdot 10^{-3}$ . Therefore it is expected that  $L_{14} \simeq 1.6 \cdot 10^{-3}$ . This parameter is involved into the description of the branching ratio of decays  $\psi'(3685) \rightarrow J/\psi(3097) + \pi$  or  $\eta$  [10]. It is determined by the following matrix elements,

$$\left| \frac{\langle 0 | G\tilde{G} | \pi^0 \rangle}{\langle 0 | G\tilde{G} | \eta \rangle} \right|^2 = (3.6 \pm 0.9) 10^{-2} \quad (17)$$

in the assumption that the multipole expansion is valid [15]. It gives the estimation,  $L_{14} = (2.3 \pm 1.1) \cdot 10^{-3}$ . One can convince oneself that the Zweig rule prediction (16) is quite satisfactory and can be thought of as the justification of multipole expansion approach for the above decays.

## 4. Conclusion

We have established the relations between the model parameters  $I_j, j = 1, \dots, 9; M_0, \xi$  and the phenomenological coupling constants  $L_i, i = 1, \dots, 10, 14, \dots, 19$  which are typical for a wide class of QCD-inspired quark models and can be verified in the experimental data. It happens that the coupling constants  $L_{14} \dots L_{19}$  of the extended chiral lagrangian are involved in a number of Zweig-type rules and can be related to the structural constants  $L_2, L_3, L_5, L_7$  in the main large- $N_c$  order.

In the common approach to the parametrization of  $\eta, \eta'$  meson masses [2, 3, 12] one neglects the constant  $\xi \simeq 0$  (see, however, [13]). It holds also for the particular QCD-bosonization models [5]. In the assumption that  $\xi = 0$  and the susceptibility  $M_0^4$  is known we can invert the relations (8), (12), (15), (16) and evaluate the model parameters from the physical input,

$$\begin{aligned} I_1 &= L_2; \quad I_2 = L_3 + 2L_2 - 4L_5 + \frac{F_0^4}{24M_0^4} - 12L_7 - 6L_{14}; \\ I_3 &= 4L_5 - \frac{F_0^4}{24M_0^4} + 12L_7 + 6L_{14}; \quad I_4 = 3L_{14} + 2L_5 - \frac{F_0^4}{24M_0^4}; \\ I_5 &= -3L_{14} - L_5 + \frac{F_0^4}{24M_0^4}; \quad I_6 = 3L_7 + L_8 + \frac{F_0^4}{96M_0^4}; \quad I_7 = -\frac{F_0^4}{288M_0^4}. \end{aligned} \quad (18)$$

We see that the tachyonic vertices with  $I_3, I_4$  can be principally determined at a given  $M_0$ . Thus the excited pseudoscalar mesons [14] make influence on the low energy meson dynamics.

On the other hand in the Gauged NJL models [7] one can find the vertex with  $\xi \neq 0$  though it is small ( $\xi \ll F_0^2/12M_0^4$ ). When existing such a vertex modifies the mass formulae for  $\eta, \eta'$  mesons that will be considered elsewhere.

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